

Modelling Volatility of the Exchange Rate of the Naira to major Currencies

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The exchange rate between the Naira and other currencies has continued to witness variability with depreciation. This variability makes it difficult to predict returns. Against this background, this paper examines the naira exchange rate vis-a-vis four other currencies. The impact of exogenous variables in modelling volatility is considered using both the GARCH (1,1) and its asymmetric variants. Three of the four returns series showed heteroscedasticity. The results of the fitted models indicate that the majority of the parameters are significant and that volatility is quite persistent. Furthermore, the results of the asymmetric model indicate different impacts for both negative and positive shocks and shows superior forecasting performance to the symmetric GARCH.

Keywords: Exchange Rate, Volatility, Leverage Effects, Exogenous Variables, Persistence, Heteroscedasticity.

JEL Classification: C52, C87, E44, E58, F31

1.0 Introduction

Exchange rate movements and fluctuations hold a numerous converse of interest from academics, financial economists and decision makers, especially since the fall of the Breton Woods consensus of pegged exchange rates among major business nations (Suliman, 2012). Following the adoption of market determined rates on the basis of demand and supply, there has been greater variability in the prices of many financial indexes. The frequency of this variability is difficult to measure as factors contributing to these, changes from time to time and dependent on the structure dynamics associated with the market. In Nigeria for instance, a unit of the US dollar that was exchanged for between 0.6100 to 0.8938 Naira from 1981 to 1985, was exchanged for between 2.0206 to 21.8861 Naira from 1986 to 1995, 21.8861 to 132.1470

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Naira from 1996 to 2005, and 128.6516 to 157.3323 Naira from 2006 to 2013 (CBN, 2013).

In Nigeria, the transformation in the foreign exchange market has been attributed to many a constituent such as those evolving example from claiming global trade, regulate transforms in the economy and structural shifts in production. Since the adoption of the Structural Adjustment Programme (SAP) in 1986, several institutional framework and management strategies have been practiced in a bid to achieve the objectives of the exchange rate policy; from the Second tier Foreign Exchange Market (SFEM) to the Foreign Exchange Market (FEM). Following continued instability over rates, further changes were introduced. These include the Autonomous Foreign Exchange Market (AFEM), Inter-bank Foreign Exchange Market (IFEM), Dutch Auction System (DAS), the Wholesale Dutch Auction System (WDAS) and currently the Retail Dutch Auction System (RDAS) which commenced in October 2, 2013.

Despite some of these policies being employed to ensure exchange rate stability, the Nigerian currency has continued to depreciate against the major currency of the world. Previous modelling attempts had centred on the predictable component of the series (Bakare and Olubokun, 2011). Later attention shifted to the residual whereby it is assumed to be normally distributed. In modelling exchange rate volatility, several authors in developed nations have employed different specifications, ranging from the parametric standard Autoregressive Conditional Heteroscedastic (ARCH) model and its variants such as the GARCH, Exponential GARCH, Power ARCH, Threshold ARCH, Fractional Integrated GARCH, etc. to its nonparametric counterparts; Kernel's, Fourier Series and Least Squares Regression.

In Nigeria however, the modelling of financial time series derivative like the exchange rate has only gained a few literature. Some of these studies have investigated the effect of exchange rate on trade as indicated by Aliyu, (2009); Ogunleye, (2009); Ayodele, (2014) and others on modelling volatility with emphasis on the empirical distribution of residuals (see Olowe, 2009; Adeoye and Atanda, 2011; Nnamani and David, 2012; Bala and Asemota, 2013; Musa, *et al.*, 2014; Sule and Bashir, 2014). In the later, some of the authors have assumed the distribution of the residuals to be normal: this is contrary to the argument that many financial time series are non normal. Also, others have specified the conditional variance as a function of only the previous day's shock and volatility.

Bera and Higgins (1993) opined that the adequate designation of the variance equation is essential as the accuracy of forecast intervals depends on selecting the variance function which correctly relates the later variances to the present data set. Also, an incorrect functional form for the conditional variance can lead to an inconsistent maximum likelihood estimates of the conditional mean parameters.

In an attempt to bridge the gap in the specification of models and estimation of parameters in modelling the exchange rate volatility of the Nigerian currency, this paper, investigates the characteristics of exchange rate volatility in Nigeria and models it with exogenous variables using the standard symmetric GARCH model and four of its variants. The objectives are to (i) measure the improvement or otherwise of the specified models and (ii) determine the forecasting performance of the specified models. The currencies considered are those of the major trading partners of Nigeria, i.e. the US Dollar, Great Britain's Pounds, Euro and the Japanese Yen. The major trading partners are determined based on the volume of Nigeria's trade with those nations who own the currencies.

2.0 Literature Review

Financial time series exhibit certain characteristics such as heavy tails, persistence, long memory, volatility and serial correlation, macroeconomic variables and volatility, non-trading periods etc (Mandelbrot (1963); Fama (1965); Black (1976); LeBaron (1992); and Glosten *et al.*, (1993)). To capture these characteristics, Engle (1982) proposed the ARCH model in which variance is assumed to be a function of previous squared shocks. Despite the success of Engle's model, it has been criticised because of the difficulty involved in estimating its coefficients in empirical applications (Rydberg, 2000). This challenge was subsequently addressed in the model by Bollerslev *et al.*, (1992). Since then, different specifications of the time varying conditional variance have been conversed in the literature. For instance in Nelson (1991), the conditional variance is specified as a function of both the size and sign of the lagged innovations. Other asymmetric models that have been proposed to capture other stylized facts of financial time series data not captured by the ARCH and GARCH models include PARCH, STARCH, TARCH, etc.

Several empirical studies have adopted these models since it was first used in modelling exchange rate by Hsieh (1988). Hsieh (1989) investigated the daily

changes in five major foreign exchange rates contain nonlinearities. He observed that GARCH models can explain a large part of the nonlinearities for the five currency exchange rates observed and that the standardized residuals from all the ARCH and GARCH models using the standard normal density were fat tailed, and the standard GARCH (1,1) and EGARCH (1,1) removed the conditional heteroscedasticity from daily exchange rate movements. In a study on volatility, Lastrapes (1989), found persistence in volatility to be overestimated when standard GARCH models were applied to a series with underlying sudden changes in variance.

Similarly, the incorporation of significant events (exogenous variables) into both the mean and variance equations in estimating volatility persistence has received much attention in the scientific community. Studies like Lamoureux and Lastrapes (1990), Gallo and Pacini (1998), Flannery and Protopapadakis (2002) investigated the effects of these variables in the Stock and Foreign Exchange Markets and showed that the introduction of exogenous explanatory variables have the tendency of decreasing the estimated persistence in the specified volatility models.

In a study on leverage effect, Engle and Patton (2001) opined that positive and negative shocks are unlikely to have the same effect on volatility with regard to equity returns. This effect they noted may be ascribed to a leverage effect and a risk premium effect. To corroborate this argument, Longmore and Robinson (2004) found the effects of shock in the exchange rate to be asymmetric in modelling and forecasting exchange rate volatility dynamics in Jamaica using asymmetric volatility models. The non linear GARCH models were also found to be better than the linear models in terms of the explanatory power.

In Nigeria, Olowe (2009) modelled the monthly Naira/Dollar exchange rate volatility using the GARCH model and five (5) of its variants. With the distribution of the residual as normal, he found volatility to be persistent and the asymmetry models rejected the existence of leverage effect even though all the coefficients of the variance equations were significant. The asymmetric models TS GARCH and APARCH were also found to be the best models.

Adeoye and Atanda (2011) in assessing the volatility of the Naira/Dollar exchange rate using the Purchasing Power Parity model found non consistency in the nominal and real exchange rates for Naira/Dollar currency thereby suggesting the relevance of long term shocks in understanding the

movement in the rates. Also, using the volatility model, they found persistency of volatility in the nominal and real exchange rate for Naira/Dollar.

However, Laurent *et al.*, (2011), Erdemlioglu *et al.*, (2012) asserted that in contrast to results in equity markets, foreign exchange returns usually exhibit symmetric volatility, that is past positive and negative shocks have similar effects on volatility.

In an independent study, Nnamani and David (2012) employed the symmetric and asymmetric volatility models to study the variability in the weekly exchange rate of the Naira and that of eight other currencies. With the distribution of the residual specified as normal, volatility was found to be quite persistent in seven of the series while it is explosive in one. The asymmetrical model provided no evidence of leverage effect for all the currencies.

Bala and Asemota (2013) used monthly data on Nigeria Naira exchange rate with that of three major currencies (US dollar, European Union's Euro and the British Pounds). In their study, they specified the mean equation as a constant and a dummy variable and the variance equation as standard model with the same dummy variable. The result of the fitted models showed reduction in persistence level in majority of the models.

Musa *et al.*, (2014) and Sule and Bashir (2014) independently modelled the daily Naira/Dollar exchange rate using some symmetric and asymmetric models. The two studies specified the mean equation as a constant and the variance equation as the standard model. They both found the asymmetric models; GJR-GARCH(1,1) and TGARCH(1,1) to show the existence of statistically significant asymmetry effect and volatility persistence to be explosive.

Unlike Bala and Asemota (2013), this study uses high frequency observations, other exogenous variables: On-Net Returns, Irregular Trading Days and Policy Change Dates in both the mean and variance equations and, in addition considers the Japanese Yen; a strong international currency.

3.0 Methodology

3.1 Data

Weekly data on the Nigerian Naira exchange rate against that of four major currencies; US dollar, European Union's Euro, the British Pound and the Japanese Yen made available by the Central Bank of Nigeria at www.cenbank.org were used. The data used covers the period January, 2002 to May, 2015. The exchange rate series of the naira to the US dollar, euro and yen have 647 observations while to the Pound sterling has 645 observations. Figure 1 presents the time plots of the exchange rate of the naira. According to Gujarati (2004) and Christoffersen (2012), unstable series such as these cannot be used for further statistical inferences because of their implications. This nonstationarity necessitates the transformation of the series.

3.1.1 Transformation

The exchange rate of each series was transformed to returns. In returns estimation, there are both theoretical and empirical reasons for preferring logarithmic returns. According to Strong (1992), theoretically, logarithmic returns are analytically more tractable when linking together sub-period returns to form returns over long intervals. Empirically, logarithmic returns have much better statistical properties i.e. are more likely to be normally distributed (Christoffersen, 2012). The weekly return is defined as:

$$a_t = \ln\left(\frac{y_t}{y_{t-1}}\right) \quad (1)$$

where a_t is the exchange rate return in period t and y_t is the exchange rate in period t . The plot of the returns as shown in Figure 2 displays such characteristic such as influential observations, volatility clustering and the time varying pattern of the shocks.

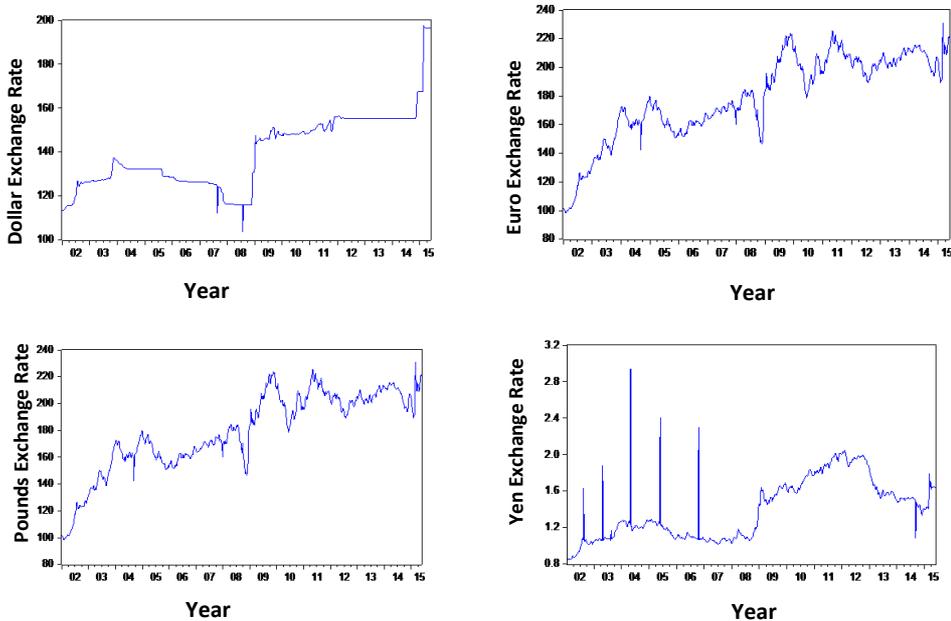


Figure 1: Series plot of Dollar, Euro, Pound and the Yen exchange rate against the Naira

3.2 Model Specification and Tests

3.2.1 Mean Equation

The conditional mean is often specified as a constant, a Moving Average (MA) process or possibly a low order Autoregressive Moving Average (ARMA) process. In this study, we have specified the mean equation as a constant, an exogenous variable; the absolute value of the difference between the present week’s returns and the previous week’s returns – On Net Returns (*ONR*) – this is to take care of the difference between a previous week’s return and the current week’s return together with an MA(1) to extract independent and identically distributed innovations. The specification of this mean equation ensures smoothening – removing the effect of unlikely events in the data set, thereby leaving whatever pattern left to be modelled by the variance equation.

$$r_t = \omega_0 + \xi(ONR) + v_t + \eta v_{t-1} \tag{2}$$

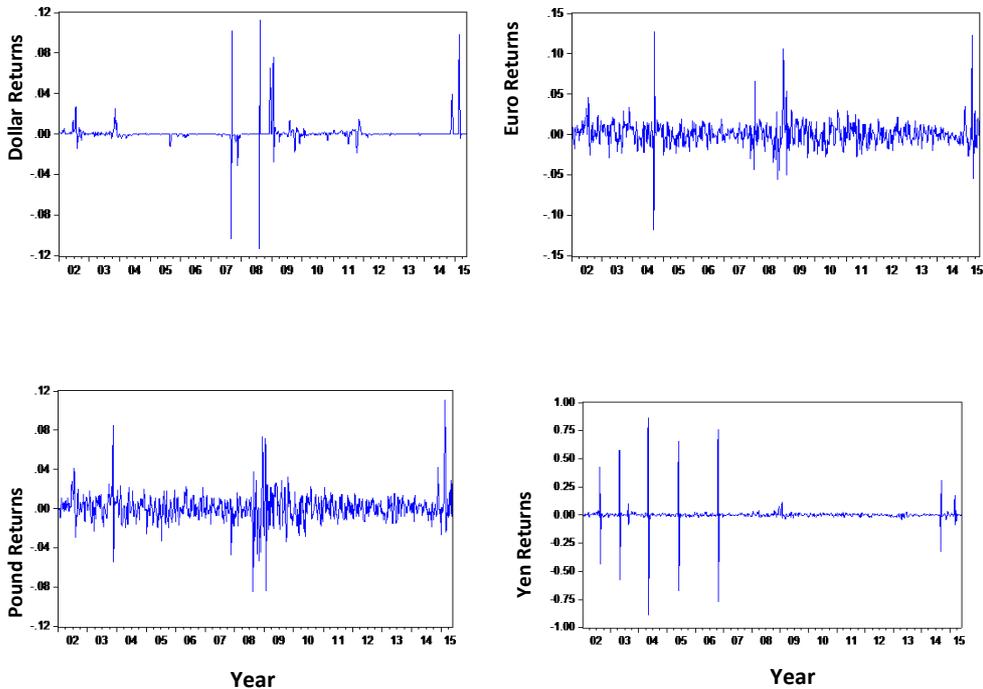


Figure 2: Stationary plot of Dollar, Euro, Pound and Yen Returns against the Naira

3.2.2 Stationarity Test

Achieving stationarity is a basic condition that must be satisfied before a model can be selected. The Augmented Dickey Fuller (ADF) test used for testing the presence of a unit root involves adding an unknown number of lagged first differences of the dependent variable to capture autocorrelated omitted variables that would otherwise, by default, enter the error term as in the regression;

$$\nabla y_t = \alpha y_{t-1} + x_t' \delta + \beta_1 \nabla y_{t-1} + \beta_2 \nabla y_{t-2} + \dots + \beta_p \nabla y_{t-p} + v_t \quad (3)$$

where $\nabla y_t = y_t - y_{t-1}$ and x_t' are exogenous regressors. The approach tests the hypotheses

$$H_0 : \phi = 1 \text{ (a series is nonstationary)} \quad \text{against} \quad H_1 : \phi \neq 1 \text{ (a series is stationary)}$$

The hypothesis is evaluated using the t statistic for ϕ

$$t_\phi = \frac{\phi^n - 1}{Se(\phi)} = \frac{\sum_{t=1}^T P_{t-1} \varepsilon_t}{\sigma^2 \sqrt{\sum_{t=1}^T P_{t-1}^2}} \tag{4}$$

H_0 is rejected in favour of H_1 if t_ϕ is greater than the tabulated critical value. When using statistical software, this is equivalent to rejecting the null hypothesis when the p-value is less than the pre-selected level of significance.

3.2.3 Testing for Heteroscedasticity

Heteroscedasticity in the returns series is a requirement for applying the GARCH model. One of the mostly used tests is the Lagrange Multiplier (LM) test proposed by Engle (1982). The procedure involves obtaining the residuals v_t from the Ordinary Least Squares (OLS) regression of the conditional mean equation. The residuals as in (2) are assumed to be ARCH(q). A straightforward derivation of the Lagrange Multiplier test as in Engle (1984) leads to the TR^2 test statistic, where the R^2 is computed from the regression of v_t^2 on a constant and $v_{t-1}^2, v_{t-2}^2, \dots, v_{t-q}^2$ as in

$$v_t^2 = \alpha_0 + \alpha_1 v_{t-1}^2 + \alpha_2 v_{t-2}^2 + \dots + \alpha_q v_{t-q}^2 \tag{5}$$

and TR^2 is evaluated against χ_q^2 .

3.2.4 Volatility Models

There are different symmetric and asymmetric models that have been employed to describe the variability in asset returns. The asymmetric models are adopted to measure the effect of both negative and positive shocks on the conditional variance. In this study, the following symmetric and asymmetric models are used;

(i) Generalized ARCH (GARCH) Model

The GARCH model proposed by Bollerslev (1986) and Taylor (1986) independently allows the conditional variance to be explained by past information (past shocks and past variances). The general model GARCH(p, q) is of the form;

$$\sigma_t^2 = \tau + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 + \sum_{j=1}^q \alpha_j v_{t-j}^2 \tag{6}$$

The parameters τ , β_i and α_j are non negative and usually estimated by method of Maximum Likelihood Estimation (MLE), while it is required that

$\sum_{i=1}^p \beta_i + \sum_{j=1}^q \alpha_j < 1$ to ensure stationarity. The model is symmetric. The most popular GARCH model in applications is the GARCH(1,1) model given in (7), Hansen and Lunde (2004) provided evidence of its suitability over other volatility models

$$\sigma_t^2 = \tau + \beta_1 \sigma_{t-1}^2 + \alpha_1 v_{t-1}^2 \tag{7}$$

In (7), weakly stationarity requires $\beta + \alpha < 1$.

(ii) Integrated GARCH (IGARCH) Model

This model was introduced by Engle and Bollerslev (1986). It imposes a constraint on the parameters of the GARCH(p,q) model by leaving out the constant term. The coefficients summing to one ensures that a shock to the conditional variance remain for all future period forecast. The model is also symmetric. The general IGARCH(p,q) model is of the form;

$$\sigma_t^2 = \sum_{i=1}^p \beta_i \sigma_{t-i}^2 + \sum_{j=1}^q \alpha_j v_{t-j}^2 \tag{8}$$

(iii) Exponential GARCH (EGARCH) Model

The EGARCH model is proposed by Nelson (1991). In this model, the natural logarithm of the conditional variance is expressed as a function of both the size and sign of the lagged residuals thereby removing the restrictions on the parameters to ensure positive variance. These allow the model respond asymmetrically to positive and negative lagged values of the corrected asset return. The model is given by;

$$\ln(\sigma_t^2) = \tau + \sum_{i=1}^p \beta_i \ln(\sigma_{t-i}^2) + \sum_{j=1}^q \alpha_j \left\{ \left| \frac{v_{t-j}^2}{\sigma_{t-j}^2} \right| - \sqrt{\frac{2}{\pi}} \right\} - \sum_{k=1}^r \gamma_k \frac{v_{t-k}}{\sigma_{t-k}} \tag{9}$$

(iv) Threshold ARCH (TARCH) Model

TARCH or Threshold ARCH and Threshold GARCH were introduced independently by Zakoian (1994) and Glosten *et al.*, (1993) to describe the asymmetry effects in financial data. The generalized specification for the conditional variance is given by:

$$\sigma_t^2 = \tau + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 + \sum_{j=1}^q (\alpha_j v_{t-j}^2 - \gamma_j v_{t-j}^2 S_{t-j}^-) \tag{10}$$

where S^- is a dummy variable that is equal to 1 when $v_{t-j} < 0$ and zero otherwise. In this form of the model, good news $v_{t-i} > 0$ has an effect of α_j on volatility while bad news $v_{t-i} < 0$ has an effect of $(\alpha_j + \gamma_j)$ provided the estimated parameter $\gamma_j < 0$. Eqn. (10) reduces to the GARCH model when $S_{t-i}^- = 0$.

(v) Power ARCH (PARCH) Model

This model introduced by Taylor (1986) and Schwert (1989) estimate the parameter of the conditional volatility. The general specification of the model is of the form;

$$(\sigma_t)^\lambda = \tau + \sum_{i=1}^p \beta_i \sigma_{t-i}^\lambda + \sum_{j=1}^q \alpha_j (|\varepsilon_{t-j}| - \gamma_j \varepsilon_{t-j})^\lambda \tag{11}$$

where $\lambda > 0$, $-1 < \gamma_j < 1$. The estimation of the parameter δ rather than it been imposed ensures the specification of the true distribution of the volatility (Longmore and Robinson, 2004).

This study used the simple GARCH (1,1), IGARCH (1,1), EGARCH (1,1), TARCH (1,1) and PARCH (1,1). Parsimonious models as these have been found to give an adequate representation of the data and outperform their complex models in both in and out of sample forecasts. The exogenous variables; ITD – irregular trading days within the week due to long holidays and PD – major policy change dates in the exchange market have also been incorporated into the conditional variance equation. In so doing, the conditional variance which makes use of information at the present time t will increase when these variables increase and decrease otherwise. In both variables, zero (0) indicate normal periods while one (1) indicate periods of change. The functional form of the conditional variance equations used in this study are as follows;

GARCH (1,1); $\sigma_t^2 = \tau + \alpha_1 v_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \mathcal{G}(ITD)_t + \psi(PD)_t$

IGARCH (1,1); $\sigma_t^2 = \alpha_1 v_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \mathcal{G}(ITD)_t + \psi(PD)_t$

EGARCH(1,1); $\ln(\sigma_t^2) = \tau + \alpha_1 \left\{ \left| \frac{v_{t-1}^2}{\sigma_{t-1}^2} \right| - \sqrt{\frac{2}{\pi}} \right\} + \beta_1 \ln(\sigma_{t-1}^2) - \gamma_1 \frac{v_{t-1}}{\sigma_{t-1}} + \mathcal{G}(ITD)_t + \psi(PD)_t$

TARCH (1,1); $\sigma_t^2 = \tau + (\alpha_1 v_{t-1}^2 - \gamma_1 v_{t-1}^2 S_{t-1}^-) + \beta_1 \sigma_{t-1}^2 + \mathcal{G}(ITD)_t + \psi(PD)_t$

$$\text{PARCH}(1,1); \quad (\sigma_t)^\lambda = \tau + \alpha_1(|v_{t-1}| - \gamma_1 v_{t-1})^\lambda + \beta_1 \sigma_{t-1}^\lambda + \vartheta(\text{ITD})_t + \psi(\text{PD})_t$$

3.3 Estimation

GARCH models parameters are estimated by maximizing the likelihood function constructed under the distribution of the residual term. The different distributions that have been assumed for this innovation are the normal (Gaussian) distribution, Student's *t*-distribution, and the Generalized Error Distribution (GED). The normal log-likelihood of parameter vector $\theta = (\tau, \alpha, \beta, \gamma, \lambda, \vartheta, \psi)^T$ is

$$L(\theta) = \sum_{t=1}^T l_t(\theta) = \sum_{t=1}^T \left(-\frac{1}{2} \ln 2\pi - \frac{1}{2} \ln \sigma_t^2 - \frac{v_t^2}{2\sigma_t^2} \right) \quad (12)$$

The maximization of (12) involves specifying the initial values of the innovation (v_t^2) and the conditional variance (σ_t^2). In this study, the conditional distribution of the innovation has been specified as a Generalized Error Distribution to capture all of the leptokurtosis present in the returns. The functional form of this distribution is

$$f_x(x; s) = \frac{se^{-\frac{1}{2}|x|^\lambda}}{\lambda 2^{(s+1)/s} \Gamma(1/s)}, \text{ where } \lambda = \left[\frac{2^{-2/s} \Gamma(1/s)}{\Gamma(3/s)} \right]^{1/2} \quad (13)$$

and the shape parameter $s > 0$. This distribution is a standard normal distribution if $s = 2$ and fat-tailed if $s < 2$. The log likelihood function of (13) is given by

$$l_n(\theta) = \sum_{t=q+1}^n \left\{ \log \frac{s}{\lambda} - \frac{1}{2} \left| \frac{v_t}{\sigma_t \lambda} \right|^\lambda - (1 + s^{-1}) \log(2) - \log[\Gamma(1/s)] - \frac{1}{2} \log(\sigma_t^2) \right\} \quad (14)$$

3.4 Model Selection

Standard selection criteria are the Akaike and Schwarz information criteria. These criteria determine the size of the errors by evaluating the log-likelihood, but also penalizes over fitting of models by including a penalty term (usually twice the number of parameters used).

This study examined the Akaike Information Criteria with the form

$$AIC = 2k + \ln\left(\frac{RSS}{n}\right) \tag{15}$$

where k = number of parameters fitted in the model, RSS = Residuals Sum of Squares and n = number of observations in the series.

3.5 Forecast Evaluation

Forecasting is an important application of time series data as such the predictive performance of the traditional forecast evaluation statistics is important in determining the appropriate model to use. The Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) are two of such evaluation statistics.

$$MAE_i = \frac{1}{N} \sum_{j=1}^{T+N} |v_{i,j+h|j}| \tag{16}$$

$$RMSE_i = \sqrt{\frac{1}{N} \sum_{j=T+1}^{T+N} v_{i,j+h|j}^2} \tag{17}$$

The model with the smallest values of the evaluation statistics is often judged to be the best model.

4.0 Results and Discussion

4.1 Descriptive Statistics of the Weekly Exchange Rate Series

Table 1 is the descriptive statistics for the returns of the four currencies. From the table, the expected returns for all series show small values. The standard deviations are also in all cases larger than the expected returns. The smaller standard deviation for the Dollar exchange rate indicates that the rate is more stable (less volatile) when compared to the Pound, Euro and Yen. This result however, contradicts Bala and Asemota (2013) where the monthly standard deviation shows the US dollar return to be the most volatile and British pound as the least volatile of the three currencies considered. Three of the four currencies present positive skewness (a right tail); excess kurtosis indicating substantial peak in the distribution (leptokurtic) is clearly observed for the weekly returns of all currencies; the JB test is also significantly large for the four returns. These indicate clear departure from symmetry

Table 1: Descriptive Statistics

Statistics	Currency			
	\$US	Euro	Pound	Yen
Mean	0.000490	0.001138	0.001421	0.000884
Median	0.000000	0.000883	0.000743	0.000173
Maximum	0.112618	0.126875	0.084740	0.866990
Minimum	-0.112704	-0.118290	-0.054462	-0.887659
Std. Deviation	0.010388	0.014936	0.013466	0.085870
Skewness	1.093927	0.706994	1.509415	-0.221031
Kurtosis	80.868280	21.063970	12.307270	72.888950
J-B Statistic	163590.1	8850.596	2572.975	131682.3
(Probability)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Observations	647	647	645	647

4.2 Stationarity and Heteroscedasticity Test

The ADF test in Table 2 for the four returns, both at 1% and 5% levels rejects the null hypothesis of the existence of a unit root in the returns.

The results of the heteroscedasticity test for the returns are given in Table 3. The null hypothesis of no ARCH effect is rejected for all currencies except for the Japanese Yen. This result was also confirmed by Nnamani and David (2012).

Table 2: Unit Root Test (ADF test statistic)

Returns	t-statistic	Critical values	p-value
\$US	-28.66116	1% level: -3.439490 5% level: -2.865464	0.0001
Euro	-23.82877	1% level: -3.439490 5% level: -2.865464	0.0001
British Pound	-19.18934	1% level: -3.439490 5% level: -2.865464	0.0001
Japanese Yen	-19.77661	1% level: -3.440291 5% level: -2.865817	0.0001

H_0 : Returns has a unit root

Table 3: Testing for ARCH Effects

Currencies	ARCH-LM	
	statistic	p-value
\$US	17.9717	0.0001
Euro	101.5764	0.0001
British Pound	67.7547	0.0001
Japanese Yen	3.6018	0.0577

H_0 : There is no ARCH effect

4.3 Analysis of Main Result

After the preliminary investigations of the data were concluded, the parameters of the appropriate model were estimated using E-Views 7, through a search algorithm that tries a number of different coefficients before converging on the optimum values.

The estimates of the coefficients of the mean equation for the fitted models are given in Table 4. For all fitted models, the exogenous variable *ONR* is found to be significant both at 1% and 5% significant levels except for GARCH (1,1) and IGARCH (1,1) models of the Euro returns, and TARCH (1,1) for the Yen returns. The significance of variables such as *ONR* shows the importance of modelling the extreme and unusual markets events that have happened during the sample period (Zivot, 2009).

Table 4: Mean Equation Coefficients:

{ Mean Equation: $r_t = \omega_0 + \xi(ONR) + \eta\varepsilon_{t-1} + \varepsilon_t$ }

Series	Model	ω_0	ξ	η
\$US / ₦	GARCH	5.75E-06* (0.000)	-0.170154** (0.015)	0.178397** (0.013)
	IGARCH	-0.000146** (0.000)	0.322750** (0.007)	0.005627 (0.010)
	PARCH	-3.19E-09 (0.000)	0.114363** (0.010)	0.133930** (0.011)
	TARCH	3.28E-06 (0.000)	0.163126** (0.022)	0.272508** (0.021)
	EGARCH	1.18E-08 (0.000)	0.095607** (0.004)	0.140811** (0.005)
	€ / ₦	GARCH	0.001425* (0.001)	-0.05373 (0.038)
IGARCH		0.01164 (0.001)	-0.02779 (0.036)	0.257965** (0.038)
PARCH		0.003569** (0.001)	-0.37537** (0.036)	0.274693** (0.035)
TARCH		0.004299** (0.001)	-0.41639** (0.033)	0.221858** (0.015)
EGARCH		0.003969** (0.001)	-0.41639** (0.033)	0.256958** (0.031)
£ / ₦		GARCH	0.001381* (0.001)	-0.08523** (0.032)
	IGARCH	0.001456** (0.001)	-0.08997** (0.032)	0.220161** (0.040)
	PARCH	0.001392* (0.001)	0.08765* (0.035)	0.247293** (0.042)
	TARCH	0.001401* (0.001)	0.08943* (0.035)	0.247467** (0.042)
	EGARCH	0.001507* (0.001)	-0.10547** (0.034)	0.239421** (0.042)
	¥ / ₦	GARCH	-1.34E-08 (0.000)	1.00000** (0.000)
IGARCH		1.48E-08** (0.000)	0.99999** (0.000)	-3.17E-08** (0.000)
PARCH		0.000301** (0.000)	0.905391** (0.006)	0.002393** (0.000)
TARCH		-5.00E-06 (0.004)	0.044339 (0.120)	-0.150288 (0.097)
EGARCH		-1.39E-08 (0.000)	1.000000** (0.000)	5.03E-07 (0.000)

Note: Numbers in parentheses are standard errors b *significant at 5% level; **significant

In Tables 5, 6, and 7 are the estimates of the parameters for the three currencies' returns.

From these tables, it can be concluded that the coefficients τ (constant), α (ARCH) and β (GARCH) are statistically significant at both 1% and 5% levels of significance for the Dollar, Euro and Pounds with expected sign for all return series. The only notable exception is the ARCH term (α) of the PARCH (1,1) model for the Euro series which is statistically insignificant even at the 5% level. Similar result was obtained by Olowe (2009) for the $\$/\text{N}$ return for fixed and managed float regime. This is in contrast with Insah (2013) where negative and positive significant values were obtained for α and β respectively.

The significance of both α and β indicates that, news about volatility (i.e., fluctuation) from the previous periods have an explanatory power on current volatility. According to Longmore and Robinson (2004), positive values for these coefficients suggests that as the market approaches expected future rate, volatility will tend to increase. In some of the models, the coefficients of α and β were high while at other times they are low. The high value of β indicates that shocks to the conditional variance are persistent while high value of α indicates that volatility adjusts quickly to changes in the market.

The leverage coefficient γ , is significant for the PARCH (1,1), TARCH (1,1), and EGARCH (1,1) in the Dollar returns, EGARCH (1,1) in Euro returns and EGARCH (1,1) in Pound returns with expected sign and positive in TARCH (1,1) in Euro returns. The negative sign and significance of γ indicates that negative shock causes volatility to rise more than if a positive shock with the same magnitude had occurred.

The parameters of the incorporated explanatory variables ϑ and ψ for irregular trading days and policy change dates respectively are found to be statistically significant for majority of the fitted models for the three returns. While the estimates of ϑ are positive, the estimates of ψ are mostly negative. This is consistent with the study of Bala and Asemota (2013) where they found estimates of ψ to be negative and responsible for reduction in persistence

The estimated persistence coefficient ($\alpha + \beta$) for the GARCH (1,1) model is calculated for all the time series. The persistence coefficient for the Euro and

Pound is less than 1 which is required to have a mean reverting process. This coefficient is greater than 1 for the Dollar.

Using the model selection criteria of AIC and the maximum log likelihood, the best three volatility models for the Dollar returns are PARCH (1,1), EGARCH (1,1), GARCH (1,1), for the Euro returns; TARCH (1,1), EGARCH (1,1), PARCH (1,1) and for the Pound returns; GARCH(1,1), PARCH(1,1), TARCH(1,1).

Table 5: Volatility Equation Coefficients with Exogenous Variables (US\$/N)

Parameter	GARCH	IGARCH	PARCH	TARCH	EGARCH
τ	9.81E-08** (0.000)		3.40E-08** (0.000)	1.52E-06** (0.000)	-2.766145** (0.368)
α	0.8477** (0.130)	0.000290** (0.000)	9.1501** (2.058)	0.5256** (0.096)	1.7760** (0.367)
β	0.2725** (0.027)	0.999710** (0.000)	0.0883** (0.021)	0.1338** (0.017)	0.7952** (0.026)
γ			0.1819* (0.091)	-0.393649** (0.098)	1.633579** (0.360)
λ			2		
ϑ	0.000117* (0.000)	4.91E-07** (0.000)	0.000515** (0.000)	0.000427 (0.000)	4.718489** (0.828)
ψ	-1.01E-07** (0.000)	-3.35E-08** (0.000)	-3.40E-08** (0.000)	-1.52E-06** (0.000)	-1.52E-06** (0.000)
$\alpha + \beta$	1.1229	1.0000	9.2384	0.6594	2.5712
Likelihood	3610.061	2752.926	3784.987	3361.669	3677.041
AIC	-11.131156	-8.488178	-11.5579	-10.36065	-11.3355
SIC	-10.06935	-8.439791	-11.4888	-10.29152	-11.2664

Note: Numbers in parentheses are standard errors. ^b* significant at 5% level; ** significant at 1% level.

Table 6: Volatility Equation Coefficients with Exogenous Variables (€ / ₦)

Parameter	GARCH	IGARCH	PARCH	TARCH	EGARCH
τ	3.71E-05** (0.000)		7.15E-05** (0.000)	7.38E-05** (0.000)	-5.65267** (1.101)
α	0.1881** (0.053)	0.1511** (0.031)	0.2396 (46.528)	-0.0272** (0.010)	0.4165** (0.104)
β	0.6235** (0.091)	0.8489** (0.031)	0.3021** (0.102)	0.2956** (0.099)	0.3898** (0.123)
γ			0.99959 (194.113)	0.933753** (0.220)	-0.46857** (0.083)
λ			2		
ϑ	0.000366 (0.000)	6.66E-05 (0.000)	0.000908** (0.000)	0.001042 (0.000)	2.363427** (0.447)
ψ	-2.95E-05 (0.000)	1.29E-05 (0.000)	-3.36E-05** (0.000)	-2.96E-05** (0.000)	-0.16507 (0.166)
$\alpha + \beta$	0.8116	1	0.5417	0.2684	0.8063
Likelihood	1940.578	1933.834	1951.432	1952.993	1952.192
AIC	-5.97088	-5.9562	-6.0013	-6.0061	-6.0037
SIC	-5.90866	-5.9078	-5.9322	-5.937	-5.9346

Note: Numbers in parentheses are standard errors. ^b* significant at 5% level; ** significant at 1% level.

Table 7: Volatility Equation Coefficients with Exogenous Variables (£ / ₦)

Parameter	GARCH	IGARCH	PARCH	TARCH	EGARCH
τ	2.51E-05** (0.000)		2.52E-05** (0.000)	2.52E-05** (0.000)	-1.61144** (1.381)
α	0.1671** (0.055)	0.1752** (0.040)	0.1676** (0.055)	0.1634** (0.062)	0.3330** (0.075)
β	0.6857** (0.072)	0.8248** (0.040)	0.6847** (0.072)	0.6839** (0.072)	0.8438** (0.040)
γ			0.008016 (0.094)	0.009232 (0.063)	-0.01375 (0.034)
λ			2		
ϑ	0.000244* (0.000)	6.97E-05 (0.000)	0.000246* (0.000)	0.000247* (0.000)	0.976041** (0.256)
ψ	-2.66E-05** (0.000)	1.15E-05* (0.000)	-2.66E-05** (0.000)	-2.66E-05** (0.000)	-0.2862* (0.115)
$\alpha + \beta$	0.8528	1	0.8523	0.8473	1.1768
Likelihood	1995.049	1982.233	1995.05	1995.053	1995.034
AIC	-6.1583	-6.1248	-6.1552	-6.1552	-6.1551
SIC	-6.09595	-6.0763	-6.0859	-6.0859	-6.08585

Note: Numbers in parentheses are standard errors. ^b* significant at 5% level; ** significant at 1% level.

4.4 Model Diagnostics

The Lagrange Multiplier’s test is used to check for heteroscedasticity in the fitted models and the ACF and PACF of the standardized residuals squared to check for the autocorrelation in the residuals. A good model should reveal no heteroscedasticity and serial correlations in the residual. The results of these tests are given in Table 8. Using the ARCH-LM test, the residuals show no heteroscedasticity left in the residual with all p-values greater than 0.01 and 0.05 for most of the models. The ACF and PACF also show no serial correlation in the residual up to the 30th lag. These results confirmed the adequacy of the fitted models.

Table 8: Diagnostics

(i) ARCH-LM Test

Series	Model	Statistic	p-value
\$US / ₦	GARCH	0.001646	0.9676
	IGARCH	0.192251	0.661
	PARCH	0.001645	0.9676
	TARCH	0.001568	0.9684
	EGARCH	0.001868	0.9655
€ / ₦	GARCH	5.334998	0.0209
	IGARCH	5.457667	0.0195
	PARCH	2.444343	0.1179
	TARCH	6.01135	0.0142
	EGARCH	0.060731	0.8053
£ / ₦	GARCH	0.624836	0.4293
	IGARCH	1.555191	0.2124
	PARCH	0.610624	0.4346
	TARCH	0.599896	0.4386
	EGARCH	0.840276	0.3593

H_0 : There is no ARCH effect in the residuals

(ii) Autocorrelation in the Standardized Residual Squared

Series	Model	AC (30)	PAC (30)	Q (p-value)
\$US / ₦	GARCH	-0.002	-0.002	0.0559 (1.000)
	IGARCH	-0.003	-0.003	0.5205 (1.000)
	PARCH	-0.002	-0.002	0.0530 (1.000)
	TARCH	-0.003	-0.003	0.1338 (1.000)
	EGARCH	-0.003	-0.003	0.1623 (1.000)
€ / ₦	GARCH	-0.008	0.001	16.549 (0.969)
	IGARCH	-0.012	-0.008	11.433 (0.999)
	PARCH	-0.012	-0.001	17.809 (0.948)
	TARCH	-0.014	0.002	23.404 (0.758)
	EGARCH	-0.012	-0.004	18.541 (0.933)
£ / ₦	GARCH	0.000	0.000	14.544 (0.988)
	IGARCH	-0.009	-0.008	10.557 (0.999)
	PARCH	0.000	0.001	14.376 (0.989)
	TARCH	0.000	0.001	14.256 (0.990)
	EGARCH	0.000	0.004	16.240 (0.973)

H_0 : There is no serial correlation in the residuals

In Table 9 are the estimates of the parameters when the exogenous variables (policy change dates and irregular trading days) are excluded from the models, suffice to say the standard GARCH models. For the Dollar returns, all the parameters are significant with expected signs at both 1% and 5% levels of

significance; for the Euro's all but the ARCH term is insignificant while for the Pound returns, only the leverage term for the PARCH (1,1) and TARCH (1,1) is insignificant.

Table 9: Volatility Equation Coefficients without Exogenous Variables (Dollar, Euro, & Pound)

Series	Parameter	GARCH	IGARCH	PARCH	TARCH	EGARCH
\$ / ₦	τ	2.99E-08** (0.000)		3.59E-08** (0.000)	3.24E-08** (0.000)	0.799399** (0.081)
	α	7.8731** (1.271)	0.1956** (0.004)	7.4884** (1.301)	0.3504** (0.049)	0.648173** (0.108)
	β	0.1048** (0.017)	0.8044** (0.004)	0.1378** (0.021)	0.4491** (0.012)	0.9501** (0.006)
	γ			0.2606** (0.072)	0.1500** (0.051)	0.5915** (0.107)
	λ			2		
€ / ₦	τ	1.27E-05* (0.000)		1.00E-05* (0.000)	9.97E-06* (0.000)	-0.676627** (0.204)
	α	0.1701** (0.045)	0.0001 (0.000)	0.1473** (0.042)	0.0649* (0.033)	0.2939** (0.049)
	β	0.7724** (0.059)	0.9999** (0.000)	0.8071** (0.045)	0.8076** (0.045)	0.9475** (0.022)
	γ			0.335847** (0.105)	0.197102** (0.064)	-0.078446** (0.029)
	λ			2		
£ / ₦	τ	9.40E-06* (0.000)		9.39E-06* (0.000)	9.39E-06* (0.000)	-0.820568** (0.237)
	α	0.1646** (0.052)	0.1129** (0.022)	0.1647** (0.052)	0.1630** (0.059)	0.3319** (0.067)
	β	0.7823** (0.058)	0.8871** (0.022)	0.7824** (0.058)	0.7824** (0.058)	0.9365** (0.024)
	γ			0.002801 (0.082)	0.003603 (0.054)	0.004543** (0.029)
	λ			2		

Note: Numbers in parentheses are standard errors ^b * significant at 5% level; ** significant at 1%

To determine the improvement or otherwise of the specified models, the volatility persistence of the specified models have been compared with the volatility persistence of the standard models. Table 10 gives the reduction (if negative) and increase (if positive) in persistence for the models.

Table 10: Estimated Persistence and Percent Reduction (Increase)

Series	Model	Persistence	
		Standard	Change
\$US / ₦	GARCH	7.9779	-85.9
	PARCH	7.6262	21.1
	TARCH	0.7995	-17.5
	EGARCH	1.5982	60.9
€ / ₦	GARCH	0.9425	-6.5
	PARCH	0.9544	-43.2
	TARCH	0.8725	-69.2
	EGARCH	1.2414	-35
£ / ₦	GARCH	0.9469	-9.9
	PARCH	0.9471	-10
	TARCH	0.9454	-10.4
	EGARCH	1.2684	-7.2

4.5 Volatility Forecast

The predictive accuracy of the models is obtained for 52 weeks out-of-sample observations and the forecast of the volatility based on the models chosen by the forecasting evaluation is given in Figure 3. The plots for the three currencies captured the sudden increase witnessed in the exchange rate between December, 2014 and May, 2015 in Nigeria. In Table 11, is the result of the evaluation of the volatility forecast using the Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) statistics. On the basis of these statistics, PARCH (1,1) is selected for the \$US/₦ and Euro’s €/₦ indexes while GARCH (1,1) is selected for the British £/₦ index.

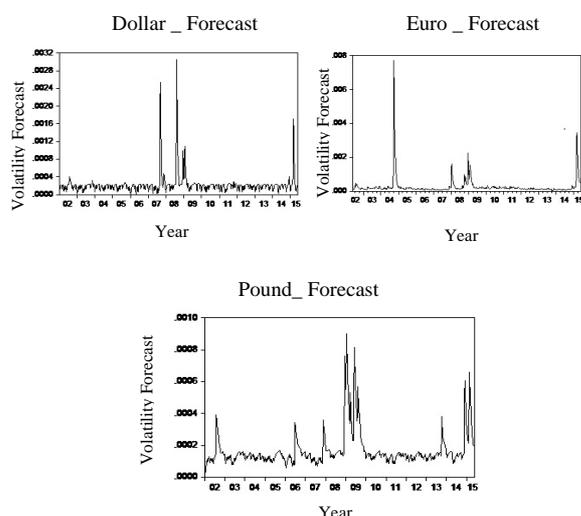


Figure 3: Volatility Forecast Plots for Dollar, Euro and Pound

Table 11: Volatility Forecasting Evaluation

Series	Model	RMSE	MAE
\$US / ₦	PARCH	0.01681	0.00472
	EGARCH	0.01695	0.00473
	GARCH	0.01957	0.00551
€ / ₦	TARCH	0.0277	0.01503
	EGARCH	0.02796	0.01504
	PARCH	0.02724	0.01456
£ / ₦	GARCH	0.02259	0.01198
	PARCH	0.02261	0.01198
	TARCH	0.02263	0.01198

5.0 Conclusion

This study examined the symmetric GARCH (1,1) model and its asymmetric variants to investigate the volatility of the Nigerian currency vis-à-vis the currency of its major trading partners. We used weekly data for the exchange rate of the four currencies; \$US against ₦, the European € against ₦, the British £ against ₦, and the Japanese Yen (¥) against ₦. The special feature of the models used in this study is that the series volatility has been modelled as a function of the standard GARCH parameters and two exogenous variables. The fitted models remove the serial correlation and heteroscedasticity in the residual. The results also showed that the conditional variance is an explosive process for the Dollar currency ($\alpha + \beta > 1$) while it is quite persistent ($\alpha + \beta < 1$) for the Pound and Euro currencies. The explosive volatility observed in the Dollar currency could be attributed to it being the dominant reserve currency in financial markets. This result is in consonance with those of other developing markets where significant persistence of volatility is observed for the US Dollar.

For the exogenous variables used in this study, while ITD tends to increase the volatility of the Naira, PD leads to a reduction in the estimated volatility persistence. This implies that the greater the number of non – trading days in a week, the more volatile the exchange rate between the Naira and the Dollar, Euro and Pound currencies. Also, the different policy change dates of the government resulted in lower volatility of the exchange rate. The forecasting performance of the fitted model in both in-sample and out-sample showed the PARCH (1,1) to have a better predictive performance for the Dollar’s and Euro’s series while GARCH (1,1) is chosen for the Pound’s series. The

asymmetric models have been found to adequately modelled the volatility of the Nigerian currency for the data used and the period under study. These results proved the assumed persistence in the exchange rate of the Nigerian currency, as such the need for proactive measures such as reduction in the number of holidays and sustainable monetary policy to cushion the effect of a volatile currency on both the nation's economy and the citizenry.

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